EE 230
Lecture 39

Data Converters

Time and Amplitude Quantization

Time Quantization

How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency is greater than twice the signal bandwidth.

This is a key theorem and many existing communication standards and communication systems depend heavily on this property

This theorem often provides a lower bound for clock frequency of ADCs

The theorem says nothing about how to reconstruct the signal

Time Quantization

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency is greater than twice the signal bandwidth.

Alternatively

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency exceeds the Nyquist Rate.

Time Quantization

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency exceeds the Nyquist Rate.

Practically, signals are often sampled at frequency that is just a little bit higher than the Nyquist rate though there are some applications where the sampling is done at a much higher frequency (maybe with minimal benefit)

The theorem as stated only indicates sufficient information is available in the samples if the criteria are met to reconstruct the original continuous-time signal, nothing is said about how this can be practically accomplished.

Review from Last Time:

Time Quantization

What happens if the requirements for the sampling theorem are not met?

Aliasing will occur if signals are sampled with a clock of frequency less than the Nyquist Rate for the signal.

If aliasing occurs, what is the aliasing frequency ?

This calculation is not difficult but a general expression will not be derived at this time. If can be shown that if f is a frequency above the Nyquist rate, then the aliased frequency will be given by the expression

$$f_{ALIASED} = (-1)^{k+1} f + (-1)^k \left[\frac{k}{2} + \frac{-1+(-1)^k}{4} \right] f_{SAMP} \qquad \text{for} \qquad \frac{k-1}{2} f_{SAMP} < f < \frac{k}{2} f_{SAMP}$$

where k is an integer greater than 1 and where f_{SAMP} is the sampling frequency

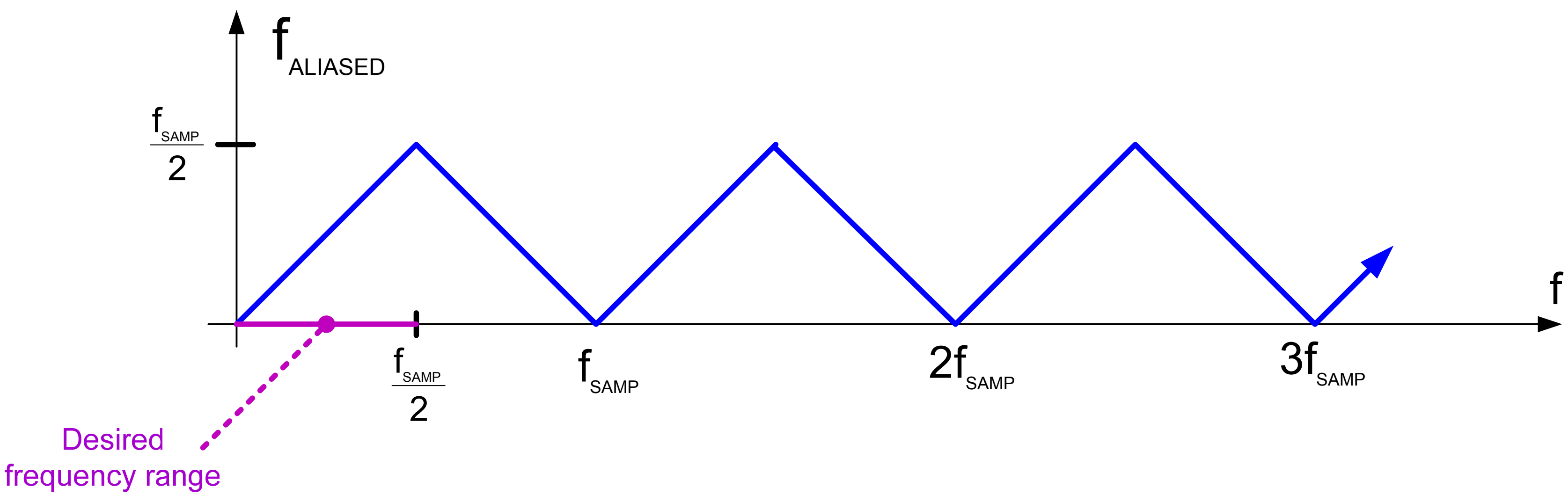
Review from Last Time:

Time Quantization

What happens if the requirements for the sampling theorem are not met?

Aliasing will occur if signals are sampled with a clock of frequency less than the Nyquist Rate for the signal.

$$f_{ALIASED} = (-1)^{k+1} f + (-1)^{k} \left[\frac{k}{2} + \frac{-1+(-1)^{k}}{4}\right] f_{SAMP} \qquad \text{for} \qquad \frac{k-1}{2} f_{SAMP} < f < \frac{k}{2} f_{SAMP}$$



Note aliased signals can not be distinguished from desired signals !

Review from Last Time:

Time Quantization

The sampling theorem and aliasing, another perspective

$$y(t)=A_0+\sum_{k=1}^{m} A_k \sin(k\omega t+\theta_k) \qquad \omega=\frac{2\pi}{T}=2\pi f$$

Consider a single period T of the band-limited signal limited to mf (where T is the period of the fundamental)

There are 2m+1 unknowns

Thus, if 2m+1 samples must be taken in the interval of length T to determine all unknowns

If these samples are uniformly spaced, the sampling rate must be

$$f_{SAMPLE}=\frac{1}{T_{SAMPLE}}=\frac{1}{\left(\frac{T}{2m+1}\right)}=(2m+1)f$$

Note this result was obtained without any reference to the sampling theorem!

How does this compare to the Nyquist rate?

$$f_{NYQUIST}=2(mf)$$

Review from Last Time:

Time Quantization

The sampling theorem and aliasing, another perspective

If a periodic signal is band-limited to mf, then the Nyquist Rate for the signal is f_{NYQ}=2mf

$$y(t)=A_0+\sum_{k=1}^{m} A_k \sin(k\omega t+\theta_k) \qquad \omega=\frac{2\pi}{T}=2\pi f$$

The Sampling Theorem (for periodic signals)

An exact reconstruction of a continuous-time periodic signal of period T from its samples can be obtained if the signal is the sampled at a frequency that exceeds the Nyquist Rate of the signal.

Furthermore, the signal can be reconstructed by taking 2m+1 consecutive samples and solving the resultant 2m+1 equations for the 2m+1 unknowns $<A_0, A_1, \ldots A_m>$ and $<\theta_1, \theta_2, \ldots \theta_m>$ and then expressing the signal by

$$y(t)=A_0+\sum_{k=1}^{m} A_k \sin(k\omega t+\theta_k) \qquad \text{where} \qquad \omega=\frac{2\pi}{T}=2\pi f$$

Time Quantization

Sampling Theorem

(continued) Aliasing

- Anti-aliasing Filters
- Analog Signal Reconstruction

Time Quantization

How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

If the signal is not band-limited, there will be insufficient information gathered in any sampled sequence to completely represent the signal by the sampled sequence

If a signal is band-limited, the signal must be sampled at a rate that exceeds the Nyquist Rate for that signal

The sampling theorem only states that sufficient information is present in the samples if the hypothesis of the theorem is satisfied but does not tell how to reconstruct the signal.

If a signal is not band limited or if it is sampled at a frequency below the Nyquist Rate, higher-frequency components will be aliased into lower frequency regions

If the energy in a signal at frequencies above the effective Nyquist Rate as determined by a sampling clock is small, the aliased high-frequency components will be small as well

Time Quantization

How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

Often the information of interest in a signal is band-limited even though the signal is not band limited.

Can this information be extracted by sampling?
(That is, can the signals of interest be reconstructed from an appropriate number of samples?)

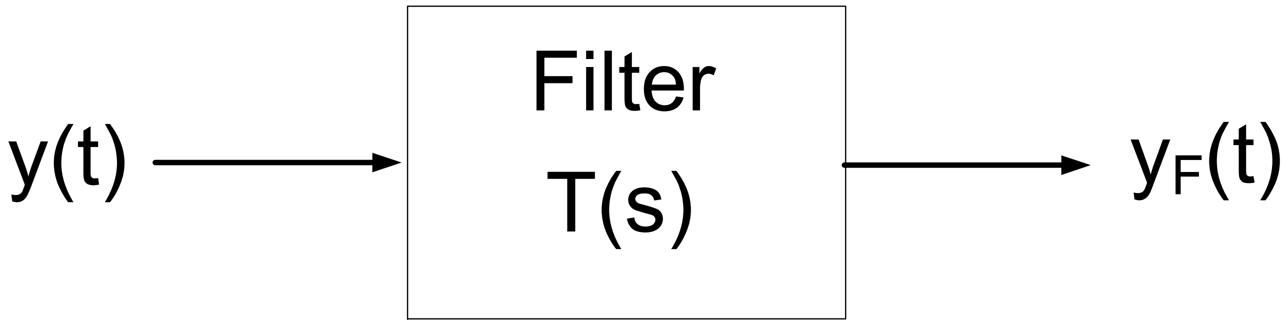
Time Quantization

Sampling Theorem

- Aliasing
- Anti-aliasing Filters
- Analog Signal Reconstruction

Time Quantization

Anti-aliasing Filters



From Laplace Transforms

$$Y_F(s) = Y(s)\,T(s)$$

From Fourier Transforms

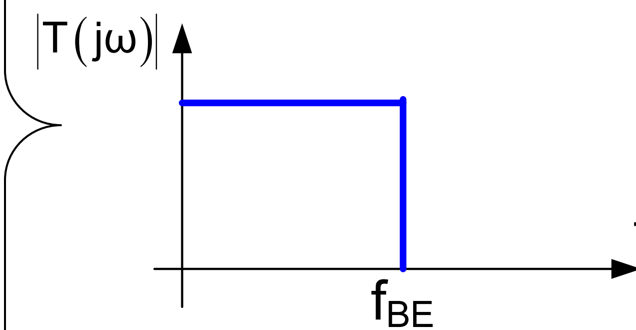
$$Y_F(\omega) = Y(\omega)\,T(j\omega)$$

From Fourier Series

$$y(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k)$$

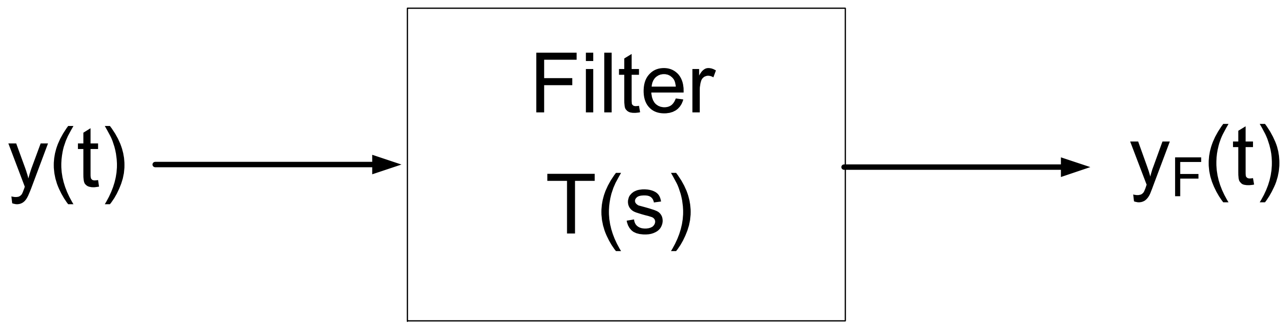
$$A_{kF} = A_k \left|T(jk\omega)\right|$$

What would an ideal lowpass filter do?



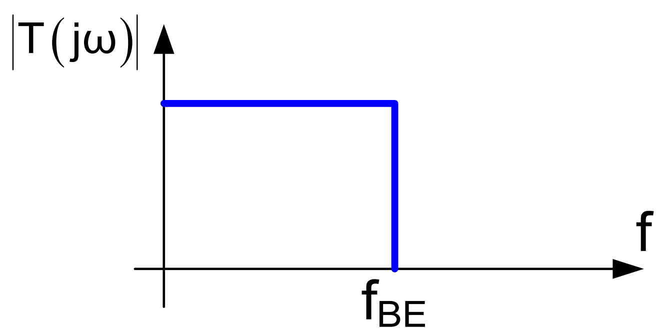
Time Quantization

Anti-aliasing Filters



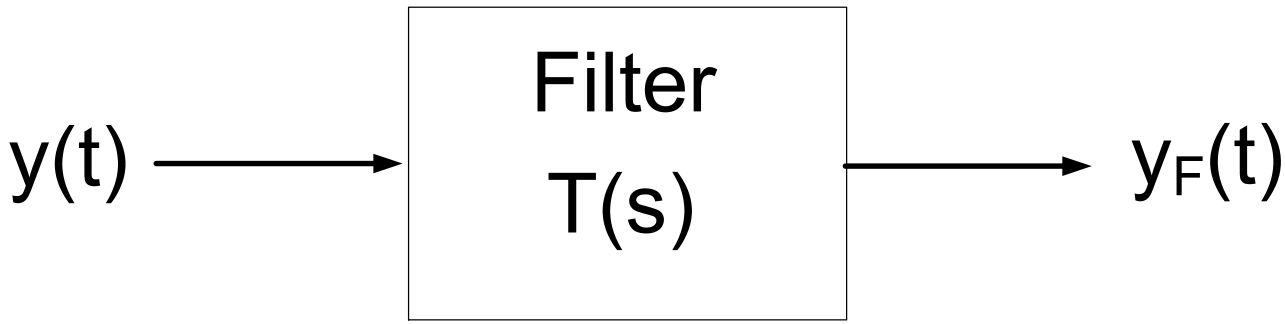
If T(s)=0 for $f > f_{BE}$ then $y_F(t)$ is band-limited

If T(s) is an ideal lowpass function with band edge f_{BE} and y(t) is either not band-limited or band-limited with a signal bandwidth that is larger than f_{BE}, then $y_F(t)$ is band-limited with signal bandwidth f_{BE}.



Time Quantization

Anti-aliasing Filters



Lowpass filters are widely used to limit the bandwidth of a signal y(t) to the band-edge of the filter before the signal is sampled.

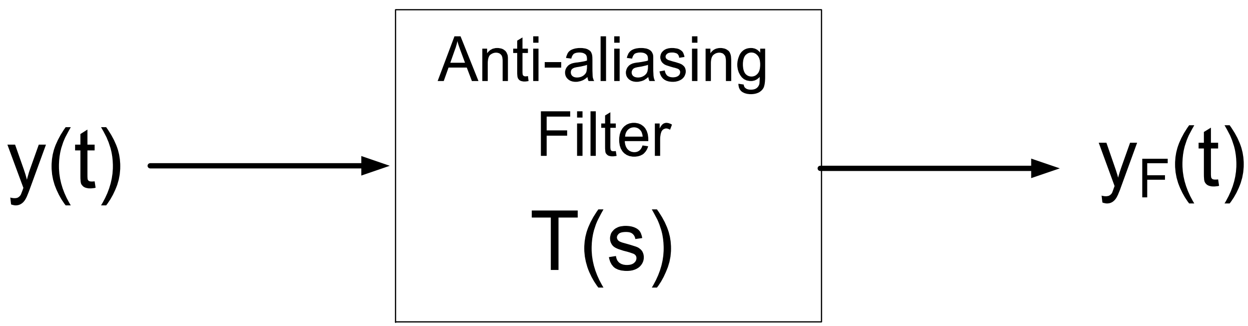
Lowpass filters that are used in this application are termed “Anti-aliasing” filters

Although the ideal lowpass filter function can not be implemented, lowpass filters with varying degrees of sharpness in the transition are widely available and well-studied. Some filters that are used for anti-aliasing filters include Butterworth, Chebyschev and Elliptic filters of varying order depending upon how Steep of a transition form the passband to the stop band is required.

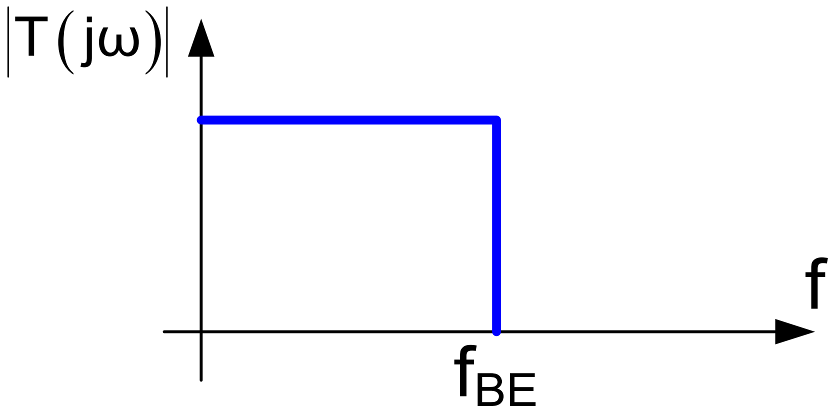
But remember that if there is information of interest above the band edge of the filter, it will be lost

Time Quantization

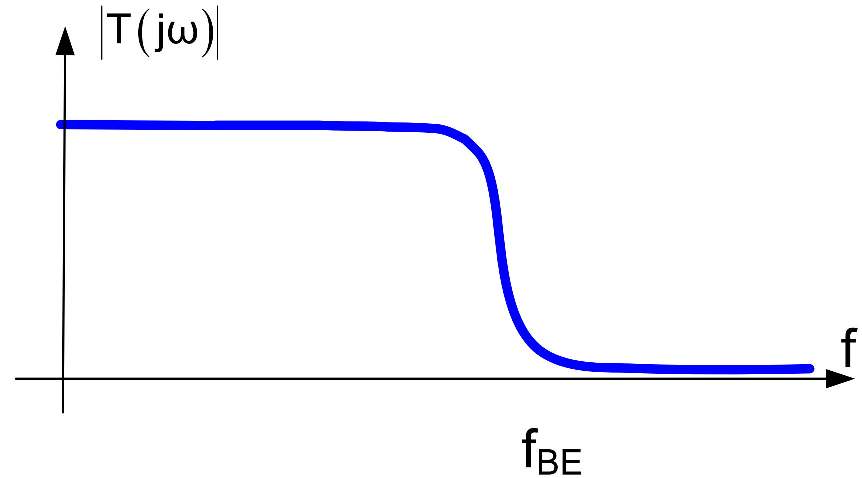
Anti-aliasing Filters



Ideal anti-aliasing filter

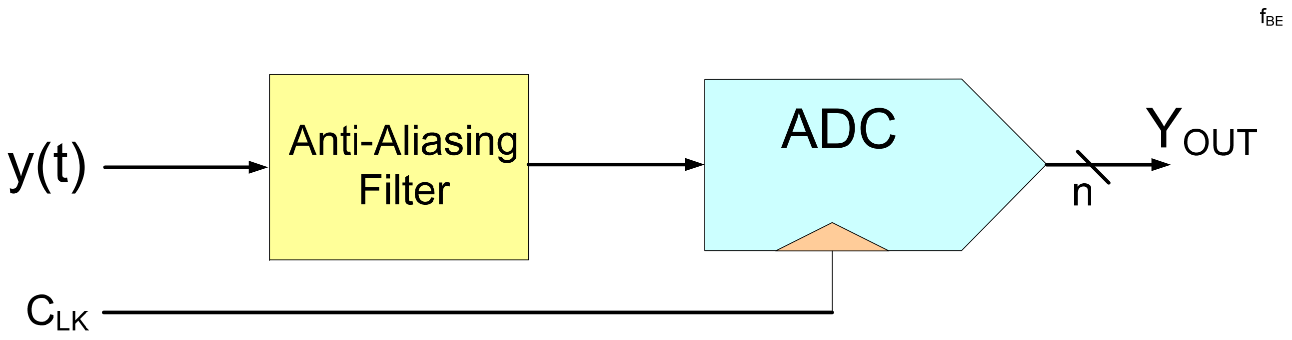


Typical anti-aliasing filter



Time Quantization

Typical ADC Environment



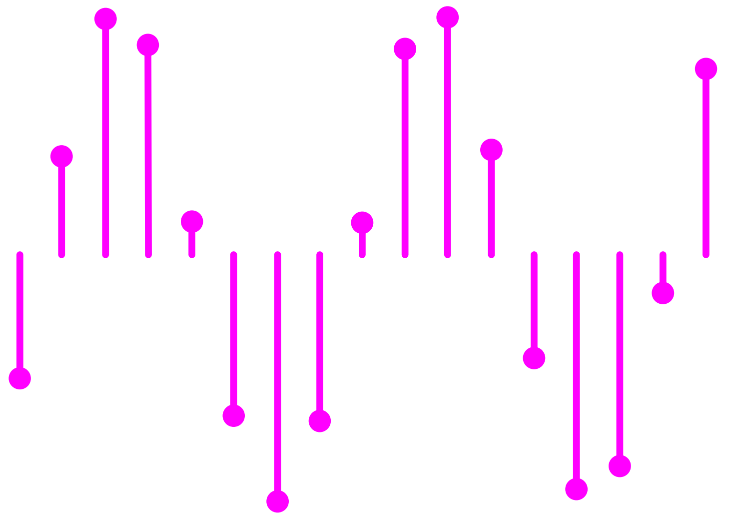
Time Quantization

Sampling Theorem

- Aliasing
- Anti-aliasing Filters
- Analog Signal Reconstruction

Time Quantization

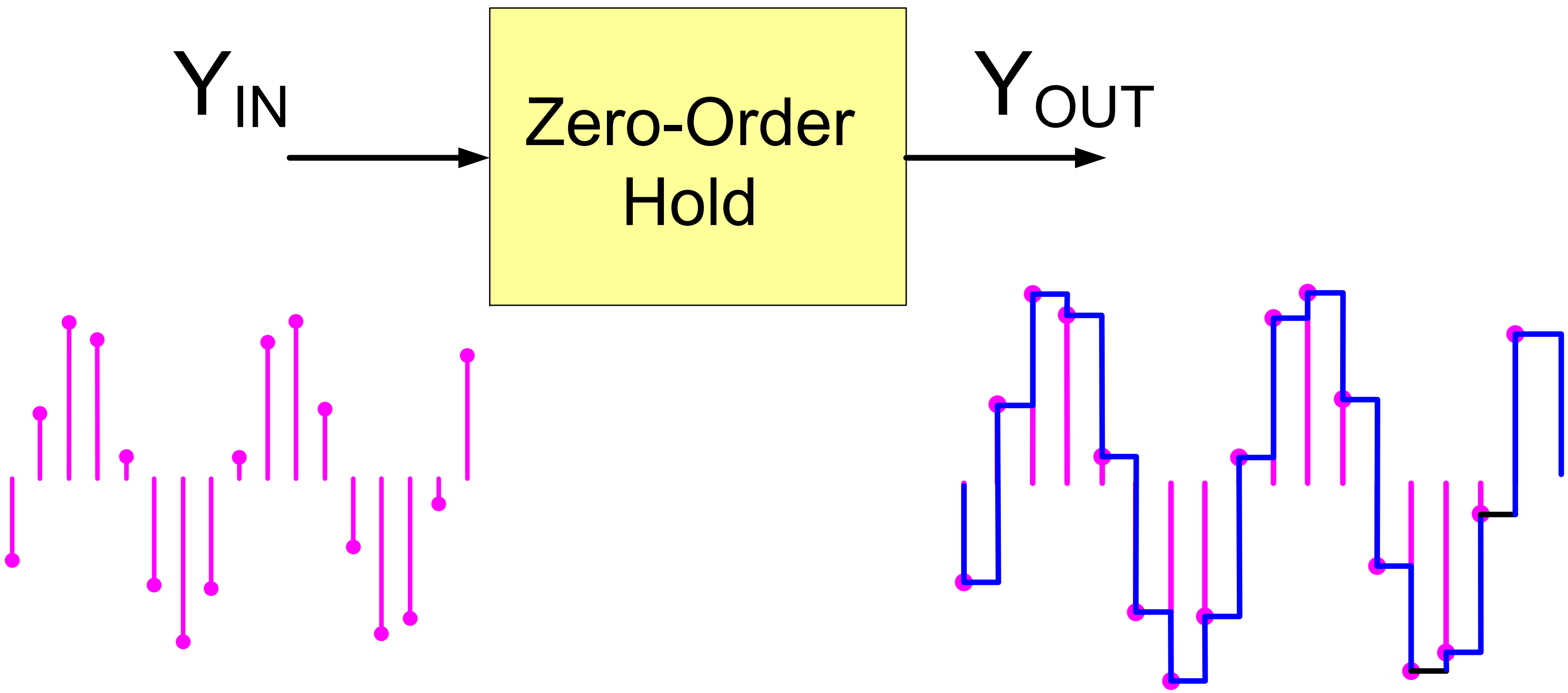
Analog Signal Reconstruction



Boolean sequence represents samples at fixed instances in time

Time Quantization

Analog Signal Reconstruction

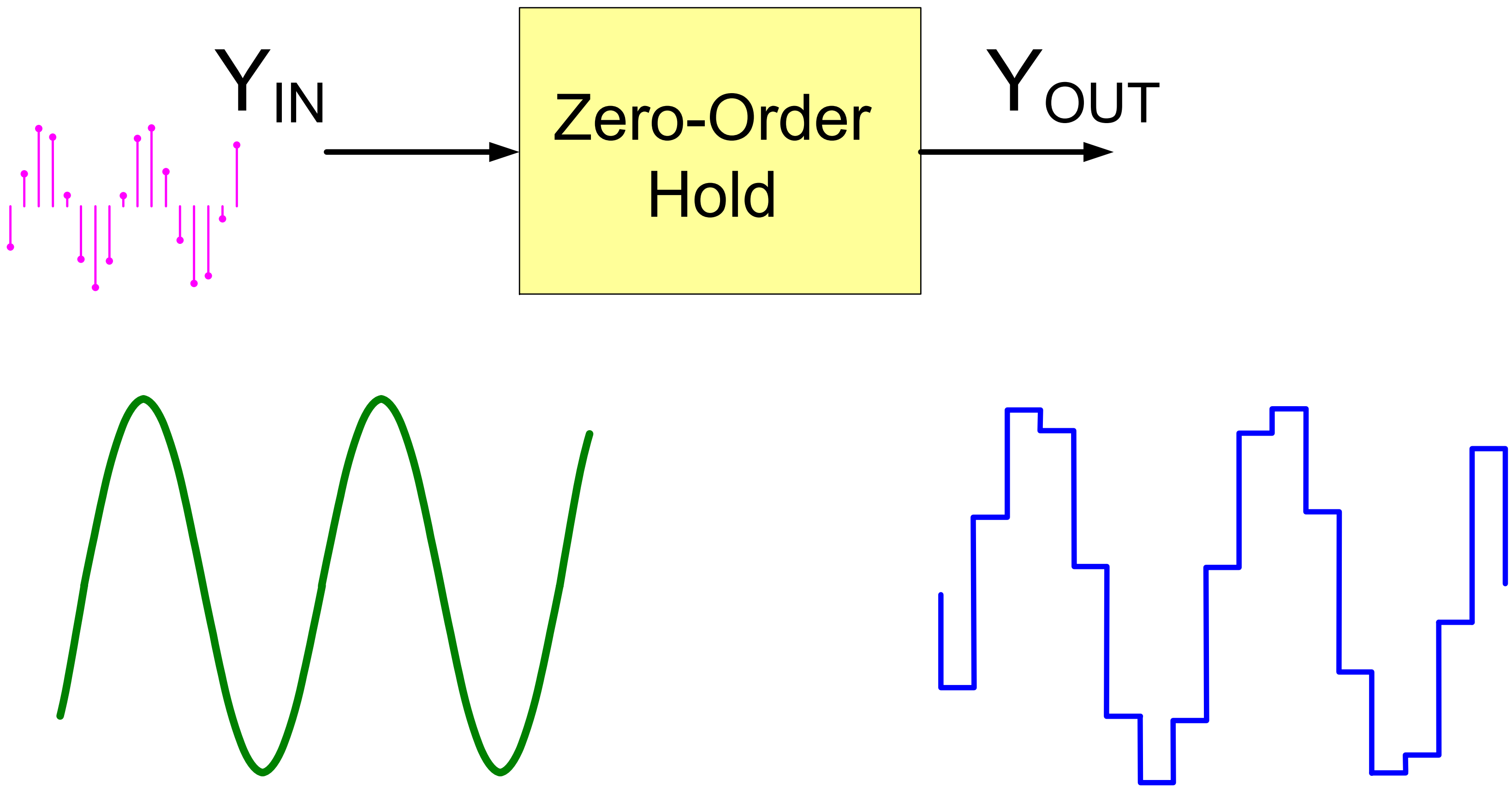


Zero-order Hold can be implemented rather easily with a DAC and other components

Although Sampling Theorem states there is sufficient information in samples to reconstruct input waveform, does not provide simple way to do the reconstruction

Time Quantization

Analog Signal Reconstruction



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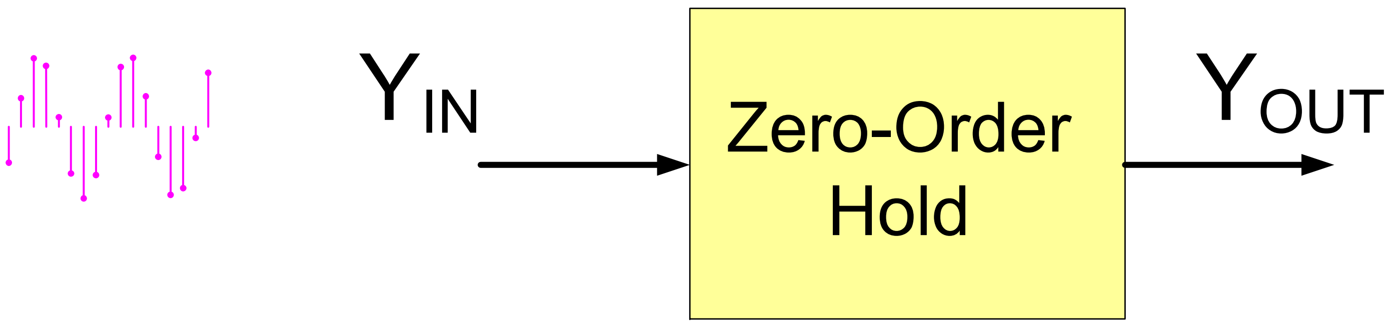
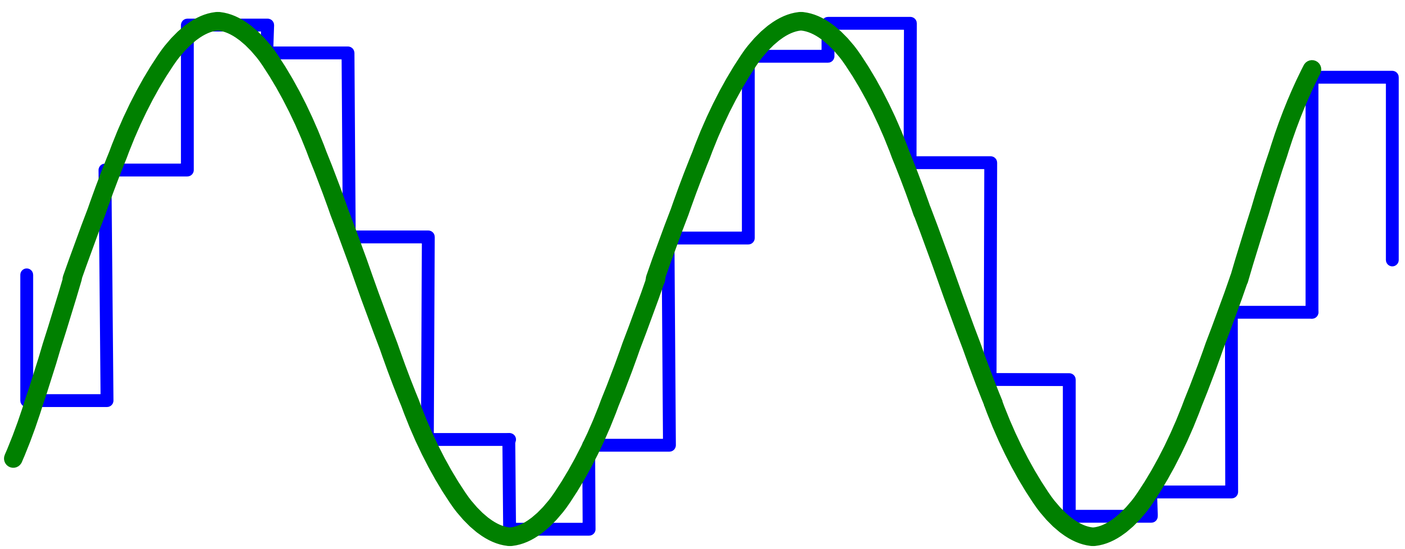
Time Quantization

Analog Signal Reconstruction

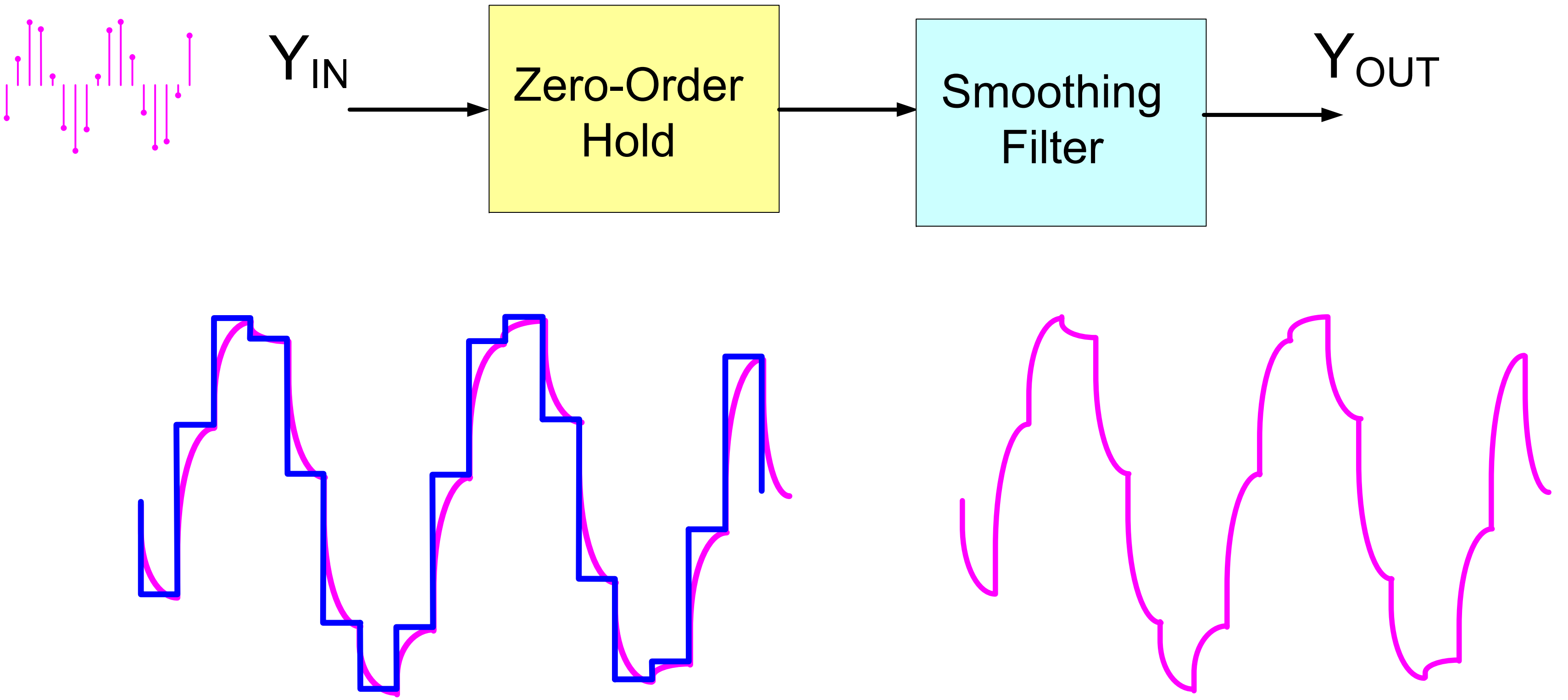




Zero-order Hold is an approximation to the actual signal that is quite close if highly oversampled but that differs considerably when sampling at near the Nyquist rate

Time Quantization

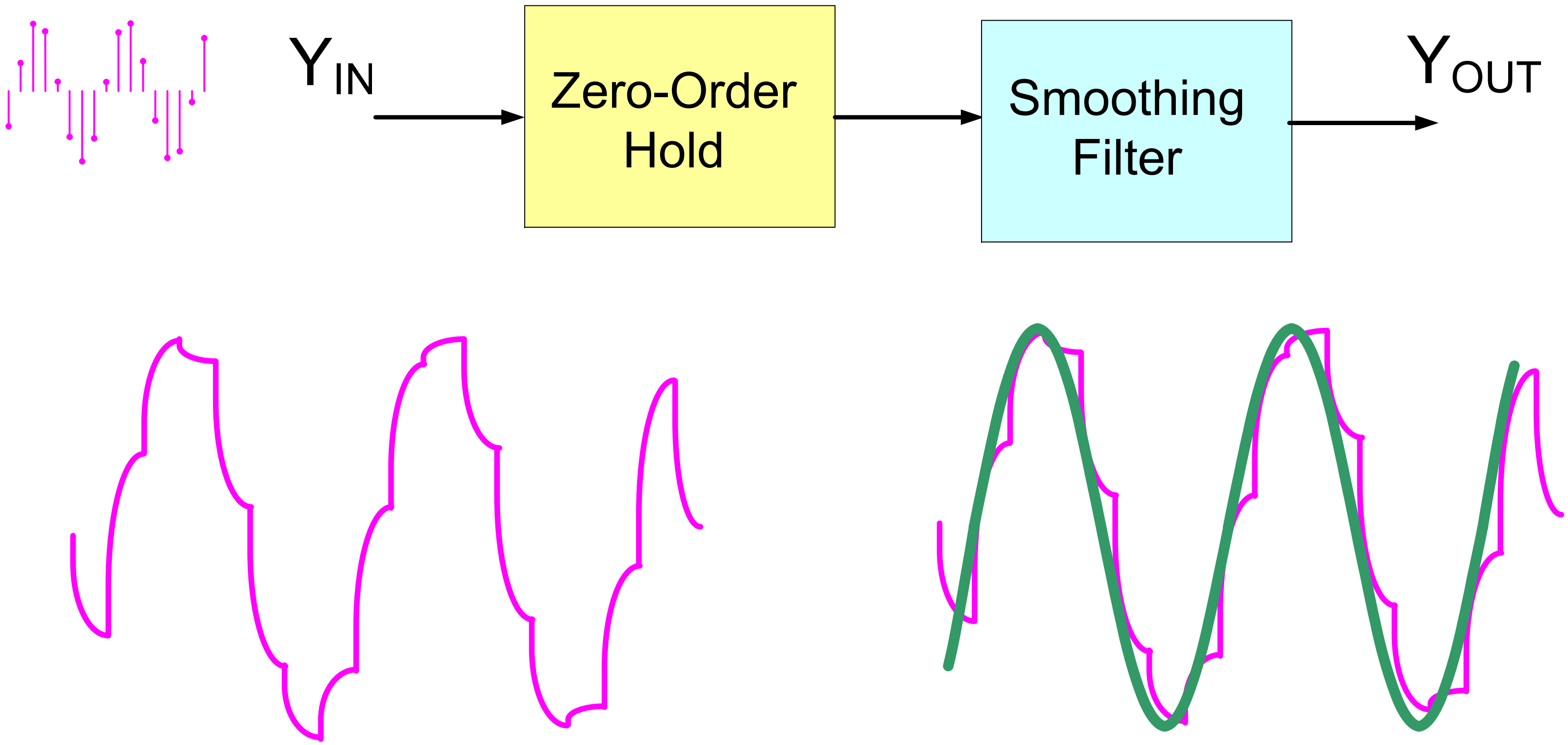
Analog Signal Reconstruction



Smoothing filter removes some of the discontinuities in the output of the zero-order hold

Time Quantization

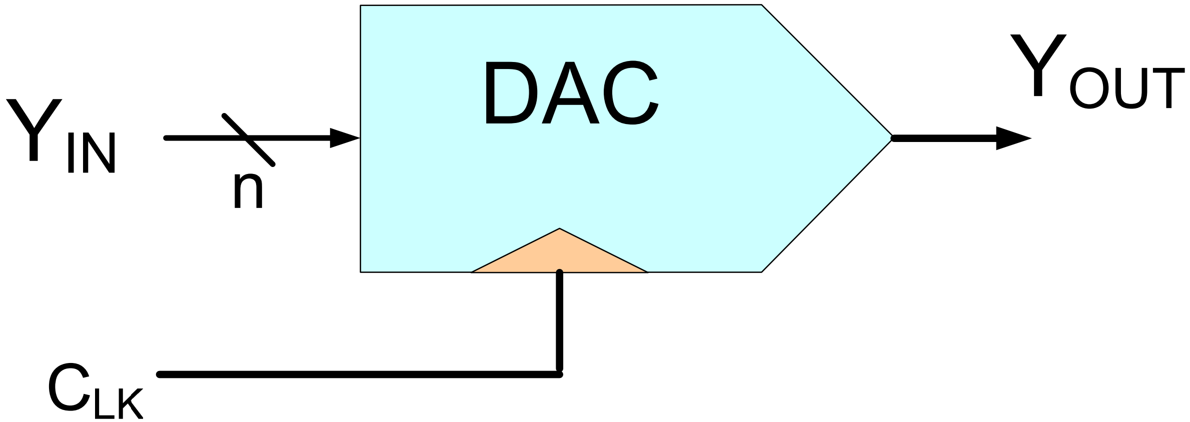
Analog Signal Reconstruction



Smoothing filter removes some of the discontinuities in the output of the zero-order hold

Time Quantization

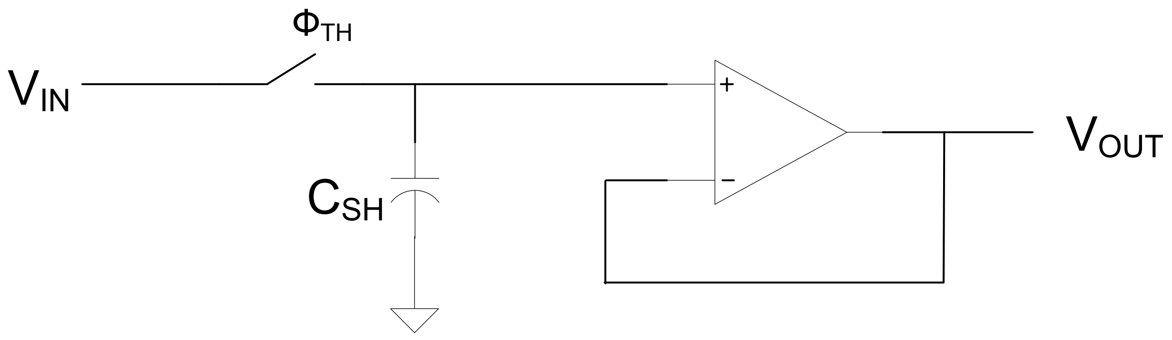
Analog Signal Reconstruction



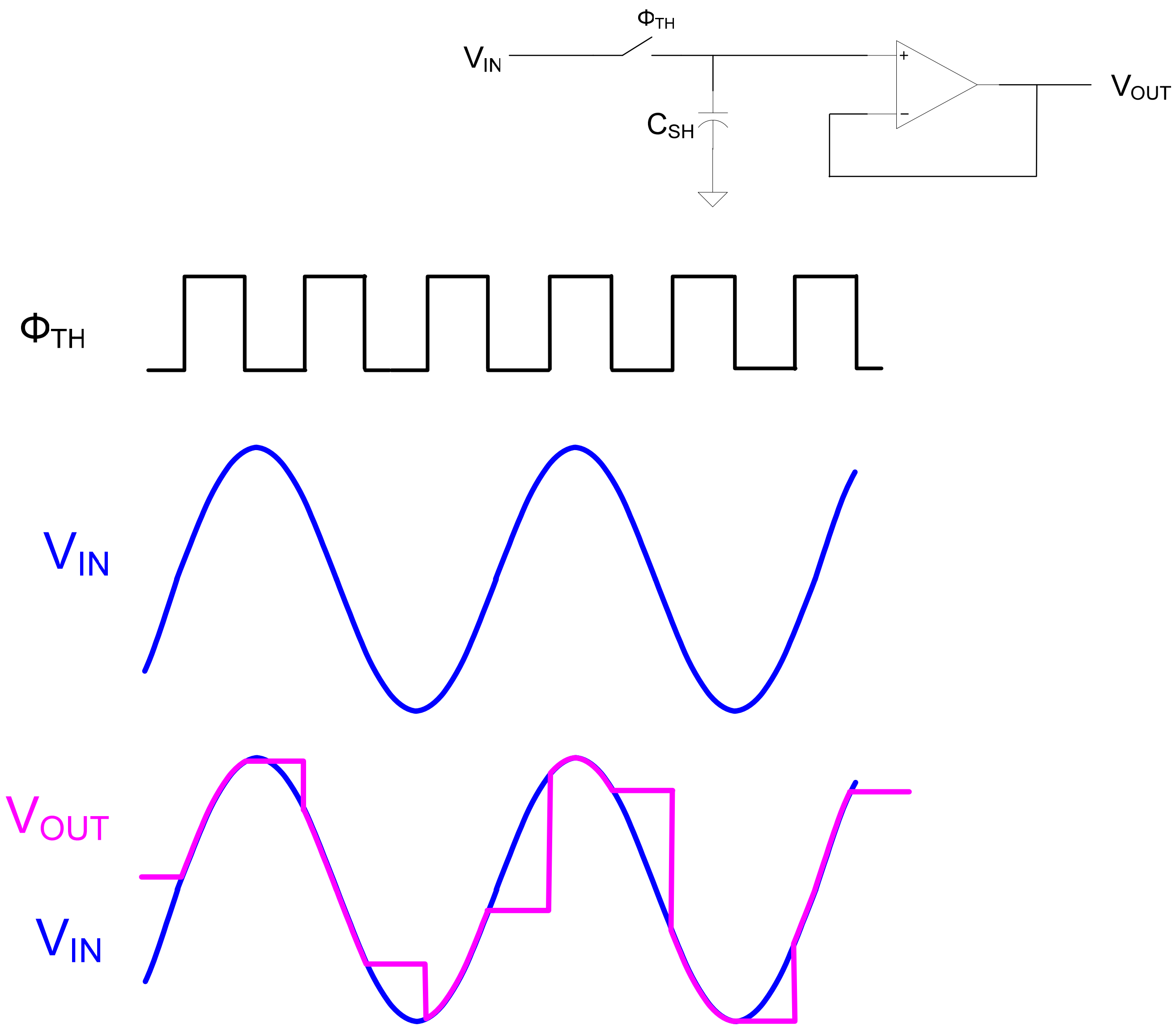
For many DACs, output only valid at some times – e.g. when clock is high

Time Quantization

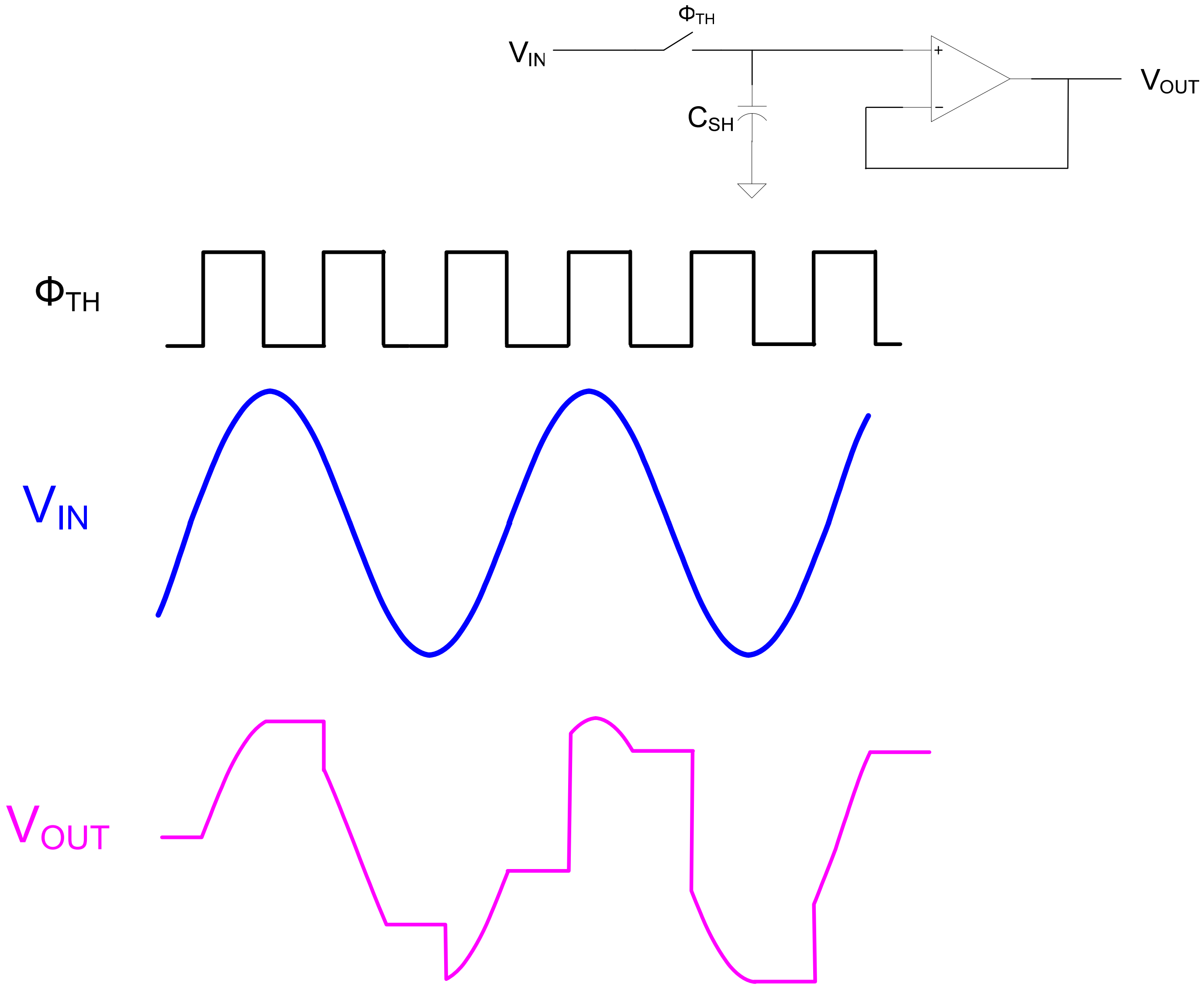
Track and Hold



Track and Hold

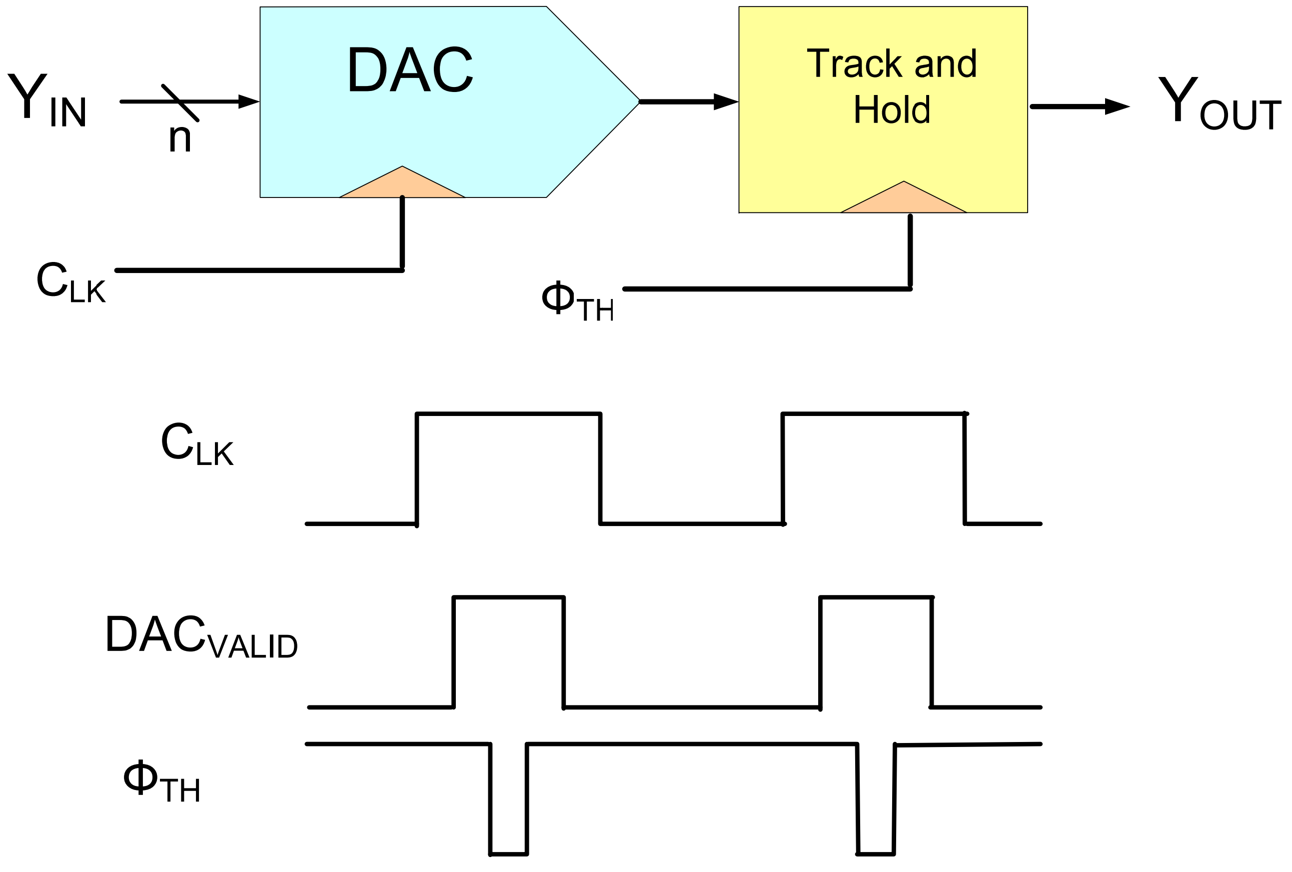


Track and Hold



Time Quantization

Analog Signal Reconstruction



- Also useful for more general DAC applications
- T/H may be integrated into the DAC

Time Quantization

Sampling Theorem

- Aliasing
- Anti-aliasing Filters
- Analog Signal Reconstruction

Engineering Issues for Using Data Converters

Inherent with Data Conversion Process

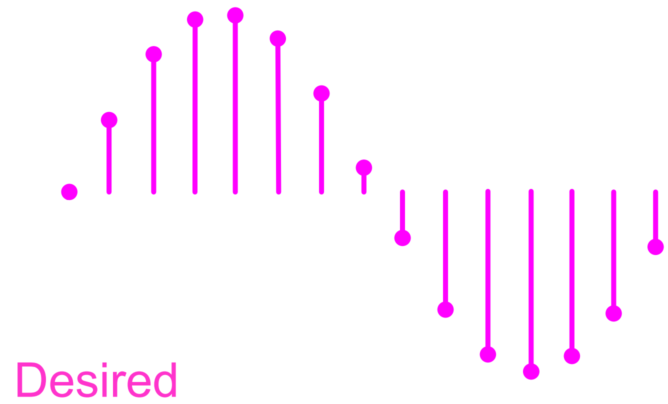
- Time Quantization

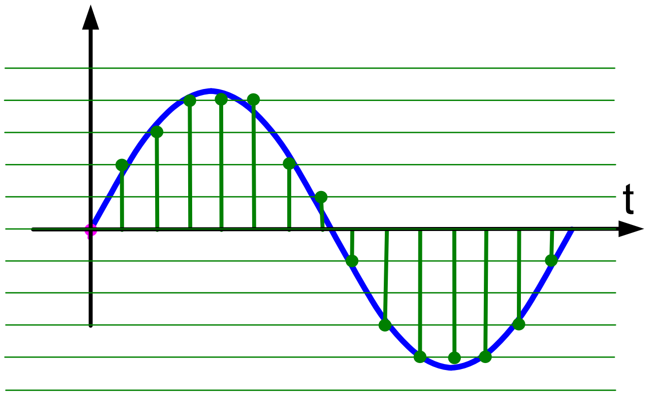
→ Amplitude Quantization

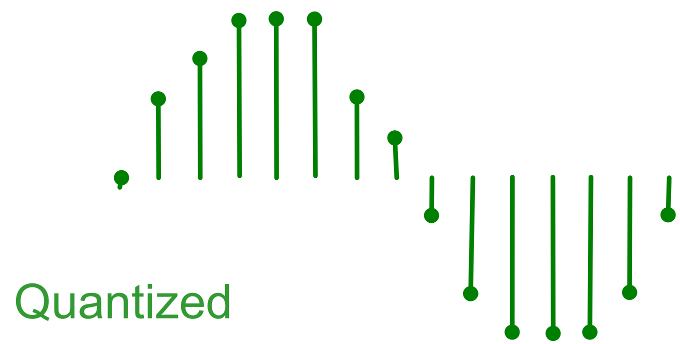
How do these issues ultimately impact performance ?

Amplitude Quantization

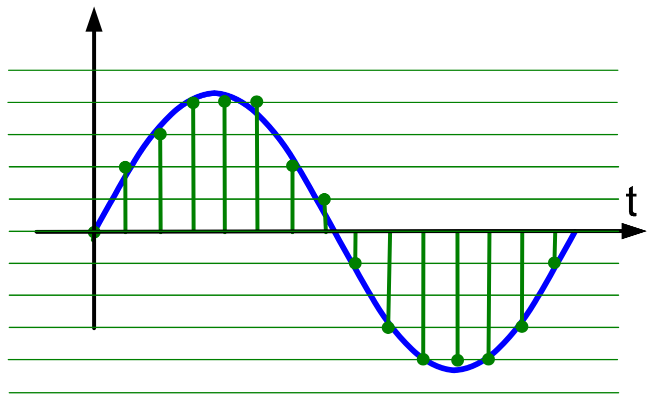
Analog Signals at output of DAC are quantized
Digital Signals at output of ADC are quantized

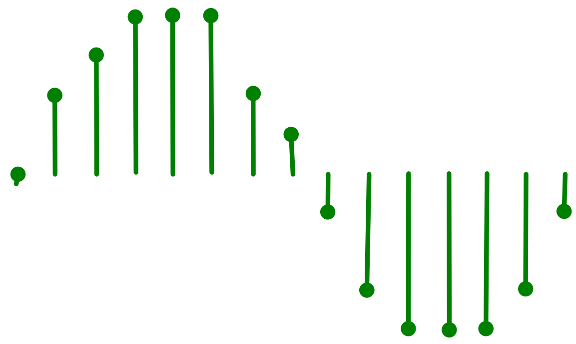






Amplitude Quantization





Amplitude quantization introduces errors in the output

About all that can be done about quantization errors is to increase the resolution and this is the dominant factor that determines the required resolution in most applications

Quantization errors are present even in ideal data converters !

Noise and Distortion

Unwanted signals in the output of a system are called noise.

There are generally two types of unwanted signals in any output

- Distortion
- Signals coming from some other sources

Amplitude Quantization

Unwanted signals in the output of a system are called noise.

Distortion

Smooth nonlinearities

Frequency attenuation

Large Abrupt Nonlinearities

Signals coming from other sources

Movement of carriers in devices

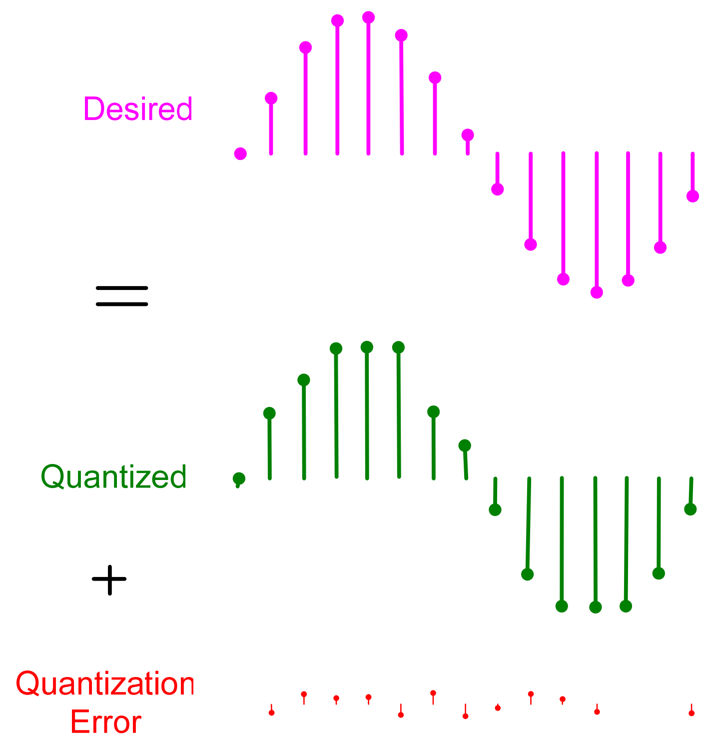
Interference from radiating sources

Interference from electrical coupling

Amplitude Quantization

Any unwanted signal in the output of a system is called <u>noise</u>

- Amplitude quantization introduces errors in the output
  - quantization error called noise



Amplitude Quantization

Unwanted signals in the output of a system are called noise.

- Distortion

  Smooth nonlinearities

  Frequency attenuation

  Large Abrupt Nonlinearities

- Signals coming from other sources

  Movement of carriers in devices

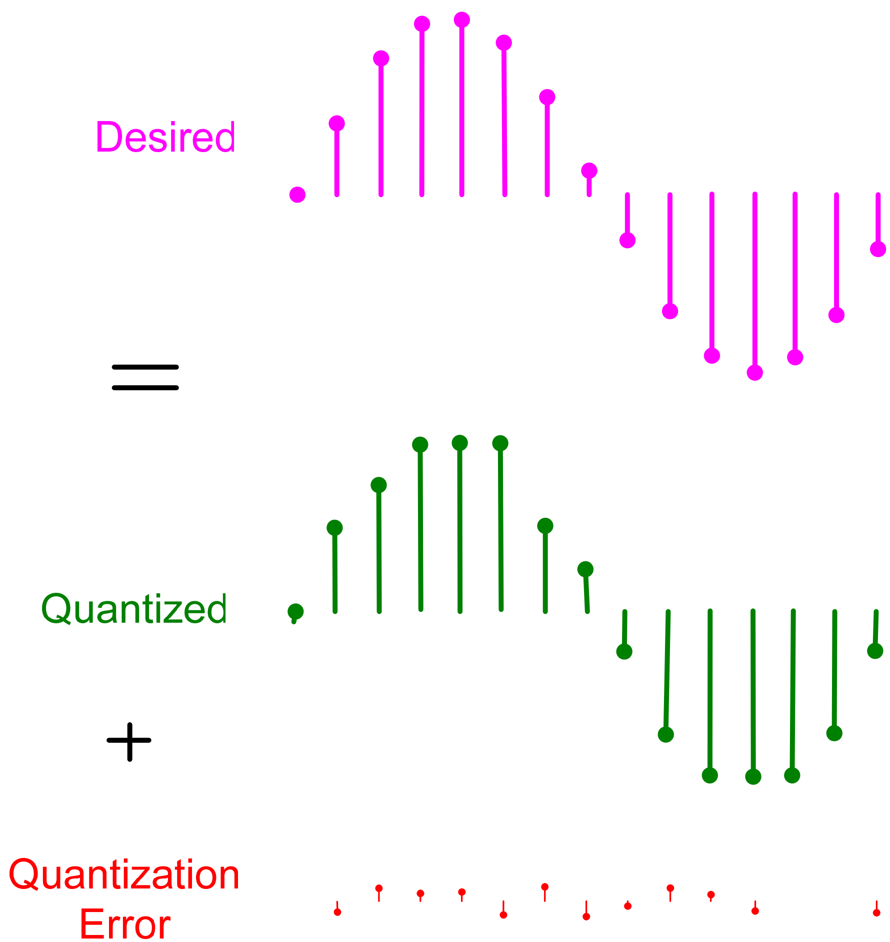
  Interference from electrical coupling

  Interference from radiating sources

- **Undesired outputs inherent in the data conversion process itself**

Amplitude Quantization

How big is the quantization “noise” characterized?



End of Lecture 39